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CALCULUS OF TORTUOSITY OF LIGHTNING CHANNEL

Luiz Dirceu Thomaz Junior[1], Moacir Lacerda[1], Robson Jaques[1], Julio Cesar Paro[2,1], Clovis Lasta Fritzen[1],
[1] Universidade Federal de Mato Grosso do Sul – Centro de Ciências Exatas e Tecnologias -
Laboratório de Ciências Atmosféricas – UFMS/CCET/LCA
[2] Instituto Federal de Educação, Ciência e Tecnologia de Mato Grosso do Sul – IFMS/Campo Grande

Abstract – In this paper we present results of comparison of lightning channel tortuosity calculated by simulation and calculated using photographic records presented in the literature. We simulated the last 1500 m of channel in several distances of cloud and under several regimen of conductivity considered as a tensor. The tortuosity of simulated channel depends on the σ_{zz} conductivity. We choose the simulated cases where the mean tortuosity angles are close to 17^o and 9^o. These angles correspond to the values presented in literature for mean tortuosity of natural and triggered lightning respectively. In these selected cases we compare the distribution of tortuosity according to height with that one calculated for natural and triggered lightning in Florida (Idone and Orvile, 1988). Some discussion about this comparison is presented.

1 - INTRODUCTION

The goal of this work is to analyze the tortuosity of lightning channels from simulated electric discharges from a four pole cloud and compare the results with the ones presented on the literature.

The method to analyze the tortousity of an electric discharge channel used in this work was proposed by Lacerda (2000). The theta angle (θ) that represents the tortuosity of lightning channel is defined as being the angle between two successive steps of the stepped leader. The steps, generated point by point in the atmosphere between the cloud and the ground, are generated using stochastic simulation. This computational process is a useful tool to simulate the physical phenomena and acquire knowledge about some characteristics of difficult measurement in certain types of irreproducible phenomena. This is the case of natural atmospheric electrical discharges (Lacerda et al. 2007).

In literature, there are several experimental and theoretical studies on the channel tortuosity generated by natural electric discharges, triggered ones or sparks produced in laboratory. These studies are presented by Hill (1968), Levine Meneghini (1977), Orvile Idone (1988), Petrarch et al. (1999), Lacerda (2000), Lupo et al. (2000), Amarasinghe et al. (2007) and Lacerda et al. (2007)

With the purpose to obtain an approximate geometry of tortuous channels we cite the models developed by Hill (1968) and Idone and Orville (1987). In the model proposed by Hill (1968), the tortuosity of the channel is analyzed using photographs of natural discharge channels. Each photograph was analyzed by a measuring microscope capable of obtaining small changes in the x and y directions, and the x axis is made to coincide with the direction of the midpoint direction of channel propagation and then is segmented into a reasonable number of parts. From the measurements of y at the end of each Δx, it is possible to measure the mean angle of deviation in the direction of the channel propagation. This method is used in nuclear physics to measure the dispersion of a particle path. Using this method, Hill (1968) obtained a mean angle of 17° for the channel tortuosity. Using the same method Idone and Orville (1988) obtained a mean value of 17° for the tortuosity analyzing photographs of natural lightning and 9° for the case of artificially triggered discharges. Lacerda et al (2007 and 2008) adapted the method used by Hill as presented in table 1, and compare results concluding that they are equivalents.

Figure 1 shows a comparison between the methods used by Lacerda et al (2007), adapted from Hill (1968) and Lacerda (2000) to determine the angles that represent the channel tortuosity.

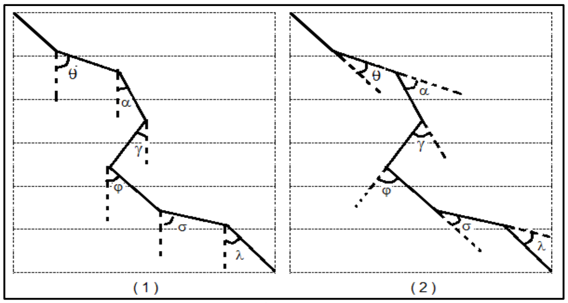


Figure 1 - (1) Representation of angles determined by Lacerda et al (2007) adapted from Hill's method (1968) to analyze the lightning channel tortuosity. (2) Representation of angles determined by Lacerda's method (2000) to analyze the lightning channel tortuosity.

2 – BASIC CONSIDERATION

The atmosphere's capability to conduct electrical current is expressed in terms of its electrical conductivity. It is an important parameter for both, good weather or storm conditions. The study of conductivity covers a wide aspect including experimental and theoretical work (MacGorman et al 1998).

In the lower and average atmosphere the conductivity is isotropic, it means that it is equal in all directions and can be determined by the product of the density and charge of the ions, as well the mobility. In the simulations of discharges analyzed in this paper, the conductivity is treated as a tensor. When the conductivity is considered a tensor, we are assuming that the conductivity acts differently in different directions, considering the troposphere also anisotropic. Near the ground under thunderstorm clouds this is the case, because there are differences between the components E_z and E_x or E_y of Electric Field ($E_z >> E_y$ or E_x). The conductivity tensor σ is represented by the conductivity matrix 3x3. The vector current density $\vec{j}$ is obtained from Ohm's Law

$$\vec{j} = \sigma . \vec{E} \qquad (2.1)$$

where ***E*** is the electric field vector produced by the charges configuration in the cloud. According to Lacerda et al (2007) Equation 2.1 can be written as

$$\begin{bmatrix} \vec{j}_x \\ \vec{j}_y \\ \vec{j}_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} . \begin{bmatrix} \vec{E}_x \\ \vec{E}_y \\ \vec{E}_z \end{bmatrix} \qquad (2.2)$$

and

$$\vec{j} - \vec{j}_x + \vec{j}_y + \vec{j}_z \qquad (2.3)$$

After that, the unit vectors $\vec{j}_x$, $\vec{j}_y$ and $\vec{j}_z$ are calculated by

$$unit\vec{j}_i = \frac{\vec{j}_i}{|\vec{j}_i|} \qquad i = x, y, z \qquad (2.4)$$

$$unit\vec{j} = unit\vec{j}_x \mid unit\vec{j}_y \mid unit\vec{j}_z . \qquad (2.5)$$

The coordinates of the next step along the stepped leaders are obtained assuming

$$P_{k+1}(x_{k+1}, y_{k+1}, z_{k+1}) = P_k(x_k, y_k, z_k) + unit\,\vec{j} * L(1+f/100)$$

(2.6)

where **L** is the length of the step, that was fixed in 50 m in the simulations, unit $\vec{j}$ is the vector calculated from Equation 2.5 and **f** is a random number ($0 \le f \le 100$). The value of *cos*θ is equal

$$cos\theta = \frac{\vec{j}_k \cdot \vec{j}_{k+1}}{j_k \cdot j_{k+1}} \qquad (2.7)$$

The angle θ that represents the channel tortuosity in the space is defined as being the 3D angle between two successive steps of the stepped leader, generated point by point in the atmosphere from cloud to ground as shown in Figure 2.

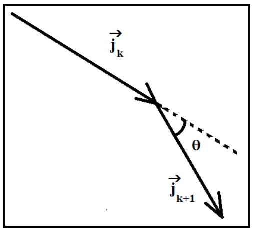


Figure 2 – The θ angle formed by two successive steps of the stepped leader (Lacerda et al. 2008).

The angles XZ and YZ, used to compare the data obtained using the simulations and those presented in literature are calculated by:

$$angXZ = arctg\left(\frac{unit_x}{unit_z}\right) \qquad (2.8)$$

$$angYZ = arctg\left(\frac{unit_Y}{unit_z}\right) \qquad (2.9)$$

These angles are formed by projecting the channel segment of the stepped leader in the respective planes XZ and YZ. Figure 3 shows the representation of these angles.

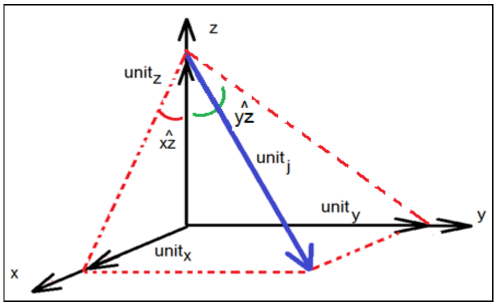


Figure 3 – The angles XZ and YZ obtained as a projection of the channel segment in the planes XZ and YZ (Lacerda et al. 2007).

The difference that may occur between the angles XZ and YZ can be understood because the simulation process is a statistical process and it also depends on the relative position of vector $unit_j$, as show in Figure 3.

The next figure presents a three dimensional simulated channel represented in the black (Lacerda et al. 2008). The blue curve is the projection of the discharge in the plane YZ, the green curve the projection in the plane XZ and the red one is the projection in the plane XY. The angle XZ used in the analysis of the channel tortuosity is obtained from the projection of the segment produced by two successive steps of the stepped leader in the channel in the three dimensions in the XZ plane. The angle YZ that can also be used to analyze the channel tortuosity is defined as the projection of this segment in the plane YZ.

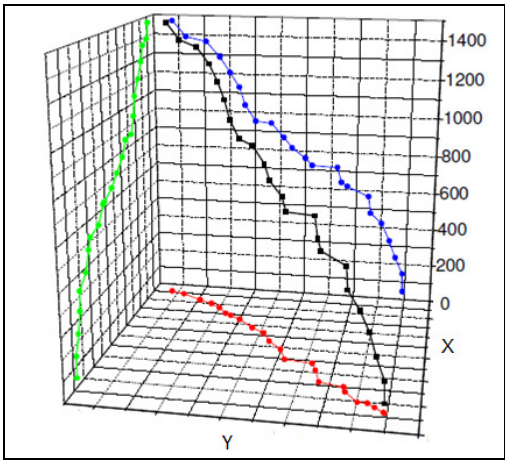


Figure 4 – Representation of a 3D simulated channel (black curve) and its projections in the planes XZ (green curve) YZ (blu curve) and XY (red curve) (Lacerda et al. 2008).

3 - DEVELOPMENTS

The simulation and calculi described in the last section were developed using the software R-simulator version 1.0. The software was created in the Atmospheric Science Laboratory (LCA) at the Federal University of Mato Grosso do Sul (UFMS).

At first, the cloud with a four-pole configuration was generated, as explained bellow. After that the discharge channels were generated and the tortuosity of the channels was analyzed.

The four-pole cloud configuration, shown in Figure 5, was based on Stolzemburg (1998a, b, c) results. The electric field generated by this cloud was simulated nearby the cloud and below it. The data were adjusted in the software up to the moment that the signature of the electric field and height were similar to the measured by Stolzemburg and collaborators.

To simulate the four-pole cloud, the positive charge distributions were generated 2800 m and 6300 m above the ground. The negative charge distributions were generated 5100m and 7100 m above the ground. Figure 5 illustrates one kind of charges distribution.

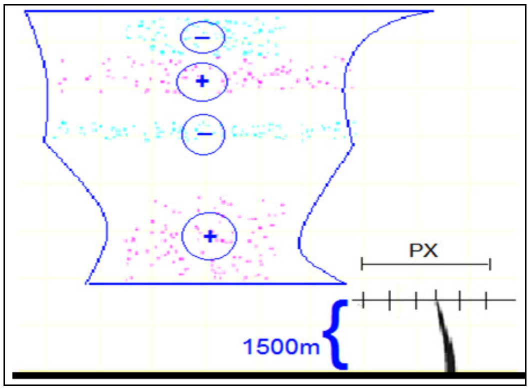


Figure 5 – Distribution of charges in the cloud configuration. The PX interval contains the points where the simulations were made. This picture is generetared by the R-Simulator software (Thomaz Jr, 2011).

After preparing the cloud, the channels were generated starting 1500 m high in several points around the cloud. This choice was made considering that this is the visible part of the channel. Depending on the point chosen for simulation, there were discharges that dissipated in the atmosphere. We considered for analysis the points where the simulations presented discharges cloud-to-ground in their totality.

Fixing the spatial coordinate Y and using steps of 460 m through the spatial coordinate X (PX in Figure 5), we simulated in each point 30 channels. The picture in Figure 5 is an example of simulation taken in the coordinates (1620 m, 3000 m, 1500 m) in the four pole cloud set in the R-simulator. Being the conductivity considered as a tensor, varying its components is the same as attributing different weights for each direction of the tensor. Using the R-Simulator it is possible to distribute the charges and describe the conductivity matrix for each round and change it automatically. Figure 6 presents the screen shown by the software R-simulator to perform one round of simulation for generating 30 channels.

X	Y	Z	q
5000	5000	5000	-1

Canais: 10

Sigma (0)			Ho	Sig0	L0	Num. Pontos
1	0	0	2000	10e-6	50	100
0	1	0				
0	0	3				

K: 9e9

Q: -0.0002
R: 1
n: 10
Hq: 3000

☐ Manter Cargas

Figure 6 – R-simulator screen containing the cloud charge configuration and the conductivity matrix.

Table 1 presents the information and description of each item found in the simulator.

	Description
X, Y and Z	Coordinates of the starting point of the simulation
q	Charge value
Sigma(0)	Conductivity matrix
H_0	scale height factor for conductivity
Sig 0	Air conductivity (scalar)
L0	Step length of the stepped leader
Num Pontos	Maximum step number of the stepped leader before cutting
Q	Ammount of charges through the channel deposited by the stepped leader
R	Channel radius
n	Number of charges in each step
Hq	Scale height (charge distribution according to height)
Canais	Number of channels in the simulation
Manter cargas	Keep or not charges through channel after every simulated channel
Simular	Start simulation
X button	Close

Table 1 – Description of the functions to be determined before performing a simulation.

After selecting the points where only discharges cloud-to-ground were found, the tortuosity for each channel was calculated taking different conductivity regimen. After each simulation some preliminary analysis were made:

A) The mean XZ angle according to PX variation describing the deviation presented by channels related to the variation of the conductivity matrix. For this analysis only the XZ was taken.

B) To simplify the notation of the conductivity matrix 3x3 it was presented as a vector 1x9. The conductivity matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & K \end{pmatrix},$$

for example, is written as (1,0,0,0,1,0,0,0,K). The mean XZ and mean YZ angles according to the variable K in the conductivity matrix considering the matrices (1,0,0,0,1,0,0,0,K), (1,5,0,5,1,0,0,0,K) and (1,10,0,10,1,0,0,0,K). These matrices describe the angles variation according to the integer K (1 ≤ K ≤ 10) that corresponds to $\boldsymbol{\sigma}_{\mathbf{zz}}$ as in Equation 2.2. The values of $\boldsymbol{\sigma}_{\mathbf{xx}}$ and $\boldsymbol{\sigma}_{\mathbf{yy}}$ were equal 1.

C) After comparing the results obtained in B) with the ones presented by Hill (1968) and Idone and Orvile (1987) the cases where the mean XZ or the mean YZ angle was similar to 17° or 9° the θ angles were analyzed according to the channel height.

4 - RESULTS

Following the procedures described in Section 3, in this section the results for the simulated discharge channels are presented.

Figure 7 presents the mean XZ angle for the conductivity matrix (1, 5, 0, 5, 1, 0, 0, 0, K) for all the values that K can assume. PX values first point is X=700 m and then it increases 460 m after each simulation up to X=3000 m. Each point in the graphic corresponds to the mean XZ angle of 30 simulated channels.

Using the same conductivity matrix, Figure 8 presents the mean XZ angle and the mean YZ angle as a function of the variable K. The red and blue bars present the mean values of each angle and their deviation (Média XZ+dp / Média YZ+dp). The horizontal lines are used do indicate the values of 9 and 17 degrees found in the literature (média_Hill/Idone/Orville). For all the values that K assumed during the simulations, some mean angles satisfied the condition of being close to the values of 9 and 17 degrees.

Table 2 presents all the values of K that fit this condition. The highlighted values of PX are the cases where the mean angle was the closest for natural or triggered tortuosity channels.

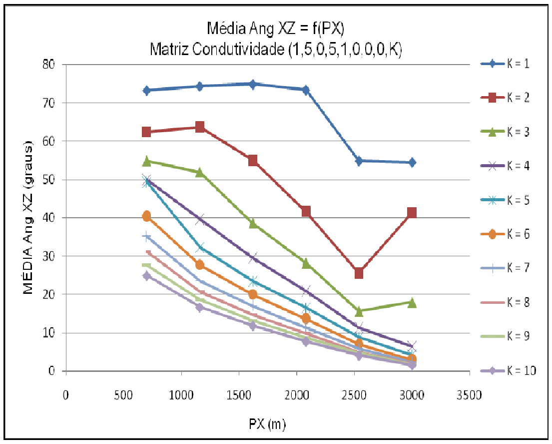


Figure 7 – Mean XZ angle (Média Ang XZ) according to PX variation using the conductivity matrix (1, 5, 0, 5, 1, 0, 0, 0, K) (Thomaz Jr, 2011).

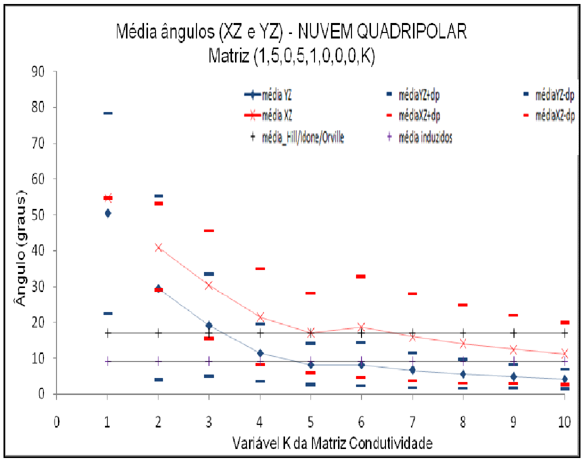


Figure 8 – Graphic comparing the mean values of XZ and YZ (Média ângulos XZ e YZ) for each value of K (Variável K da matriz condutividade) (Thomaz Jr, 2011).

Four pole Cloud					
Conductivity Matrix	Kind of discharge to be compared	K	PX (m)	Mean XZ Angle	Mean YZ Angle
(1,5,0,5,1,0,0,0,K)	Natural	5	700	49,37	24,04
			1160	32,31	12,11
			1620	23,42	6,56
			2080	**16,43**	1,72
			2540	8,7	4,64
			3000	4,08	16,02
	Triggered	6	700	40,49	16,84
			1160	27,77	**9,28**
			1620	20,03	5,02
			2080	13,65	1,05
			2540	7,03	4,31
			3000	2,92	12,78
(1,10,0,10,1,0,0,0,K)	Triggered	9	700	49,76	19,26
			1160	34,22	**9,44**
			1620	25,67	4,76
			2080	18,22	0,56
			2540	10,45	6,41
			3000	3,61	17,33

Table 2- Mean XZ and YZ angles obtained by simulations. Highlighted cells indicate the discharges chosen for analysis (Thomaz Jr, 2011).

In table 2. the differences between XZ and YZ angle is due the relative position of the plane that contains the discharge, to the plane XZ and YZ.

Figures 9, 10 and 11 present the variation of the respective case highlighted in the table according to the height (altura) in meters and for each one the variation of the theta angle.

Figure 9 represents the results for a four pole cloud where there is only one case for comparison to natural discharges. The mean XZ (2D) angle is 16,4° (K=5 and PX=2080 m). The angle and its fluctuations increase with height for XZ angle. The theta angle (3D) is lower than XZ angle.

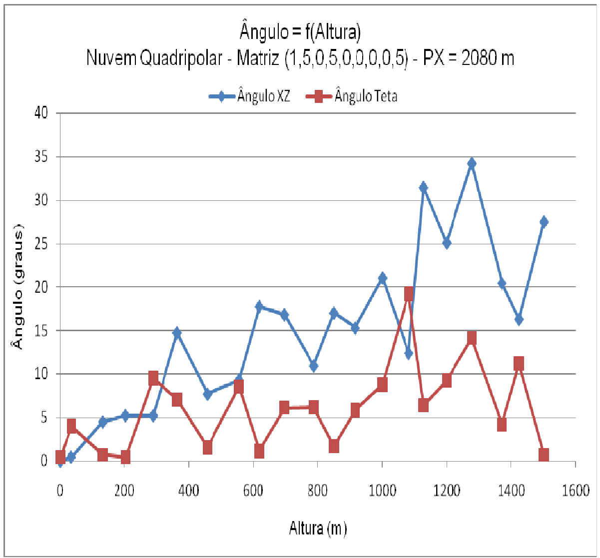


Figure 9 – Variation of the XZ (Ângulo XZ) (2D) and theta (teta) (3D) angles as a function of height (altura) in degrees (graus) (Thomaz Jr, 2011).

Figure 10 presents another analyzed case that can be compared to triggered discharges, but in this case the mean YZ angle is used. For K=6 and PX=1160 m the mean YZ angle is 9,2°. In these two cases we have σ_{xy} = σ_{yx} = 5 in the conductivity matrix.). The angle and its fluctuations increase with height for XZ angle. The theta angle (3D) is lower than XZ angle after 1000m.

The other case found on the simulated discharge channels is shown in Figure 11. What makes this case different from the other is that in the conductivity matrix σ_{xy} = σ_{yx} = 10, K=9 and PX=1160 m, and both 3D and 2D angles are close to the same values. YZ angle is 9,4° that is close to the tortuosity of triggered channels.

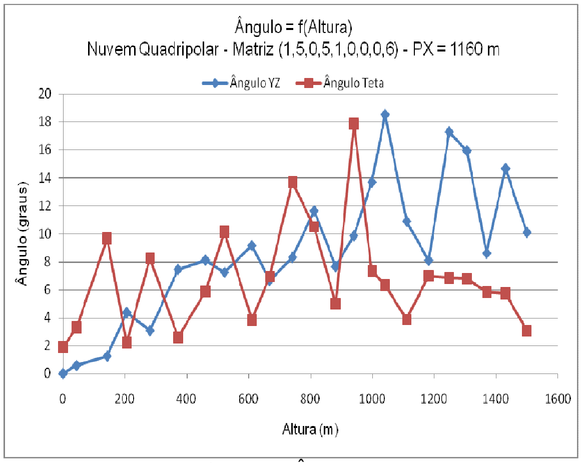


Figure 10 – Variation of the XZ (Ângulo XZ) (2D) and theta (teta) (3D) angles as a function of height (altura) (Thomaz Jr, 2011).

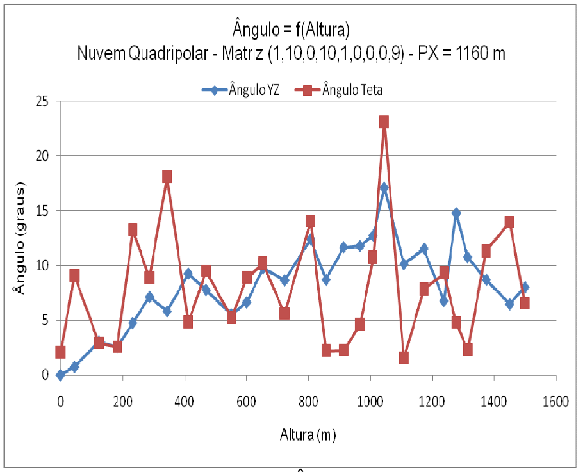


Figure 11 – Variation of the YZ (Ângulo YZ) (2D) and theta (teta) angles (3D) as a function of height (altura) (Thomaz Jr, 2011).

5 - ANALYSIS OF THE RESULTS

In this section we discuss the tortuosity of some simulated discharge channels, presented in the previous section, and compare them to the values presented in Literature.

The behavior of the XZ, YZ and θ angles for a four-pole-cloud is similar with the variation of height. This is more visible when the horizontal conductivity of the channel is high σ_{xy} and σ_{yx}.

Figures 12 and 13 present the results for θ in natural discharges and triggered ones respectively. These figures were obtained using the values described by Idone e Orville (1988).

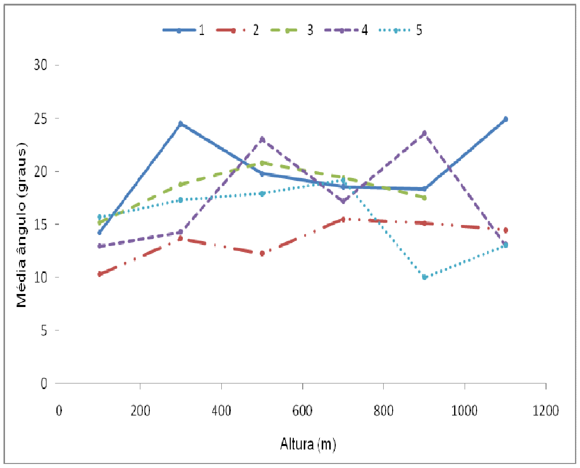


Figure 12 – Mean θ angle (Média ângulo) (2D) as a function of height channel (Altura) for five natural discharges. This figure was adapted of Idone and Orville (1988) data.

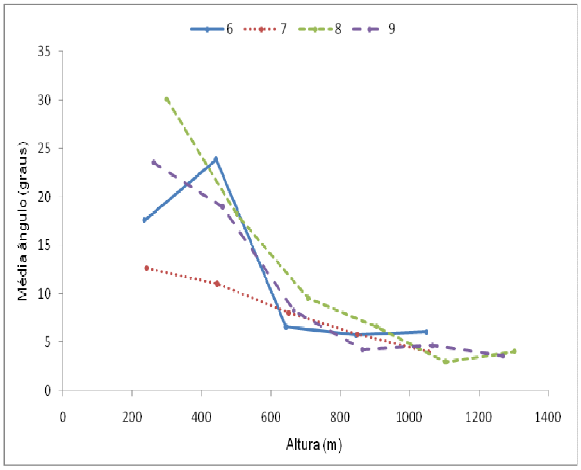
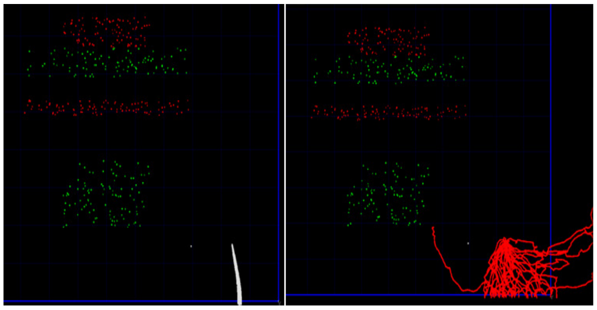


Figure 13 – Mean θ angle (Média ângulo) (2D) as a function of height channel (Altura) for four triggered discharges. This figure was adapted of Idone and Orville (1988).

Finally, to ilustrate the effect of the function “Manter Cargas” (keep charges) (see figure 6 and last field of talbe 1) in the shape of the simulated channels we present in Figure 14 the effect of turning this function “ON” or “OFF”. When this function is “OFF” the stepped leader generated channels close to the same region (figure 14(1)). When this function is maintained “ON” there is a big dispersion through the space (figure 14 (2)). In these run we simulate 30 channels. When this function is “ON”, initially the channels are centered around a set of cloud to ground (CG) channels. But, after a number of CG channels some of them go to the atmosphere or return to the lower positive center on the cloud base. This shows that the existence of residual electric charges (points, lines, layers) in the atmosphere may interfere in the tortuosity of channels. In case of a multiple return stroke lightning this fact (maintaining charges) means that a new channel can be formed close to the previous return stroke channel.

Comparing figures 9 10 and 11 with figures 12 an 13 we notice that the behavior with height of simulated lightning is different to those of natural and triggered lightning where the tortuosity maintains constant (natural lightning) or decrease (triggered lightning) with height. Based on analyses of figure 14 we infer that this is probably due the inexistence of charge (points, lines or layers) in the atmosphere close to the ground that probably would increase the tortuosity, during the simulations.



(1) (2)

Fig. 14 Comparison between shape of channel with the existence or not of residual charge along the channel, during simulation of 30 channels. (1) no residual charges along channel, (2) existence of residual charges. (See text for details)

6 - CONCLUSIONS

In this paper we show that the conductivity matrix really influences the tortuosity of the lightning channel. The simulations showed results where the mean 2D-tortuosity (XZ or YZ angles) of the simulated channel, similar to results obtained for natural discharges (17^o) and triggered discharges (9^o) can be generated by varying the conductivity and relative position of channel to the cloud, according to values presented in literature. The simulated cases doesn´t have similar behaviors to those angles presented in literature for natural and triggered lightning when the mean angle (XZ or YZ) are ploted as a function of height. We infer that this is probably due the inexistence of charges (points, lines or layers) in the atmosphere close to the ground that probably would increase the tortuosity, during the simulations. Finally, the 3D angle in the simulations were smaller than the 2D angle for the simulations.

7 - REFERENCES

[1] AMARASINGHE, D., SONNADARA, U., BERG, M., COORAY, V., "Channel tortuosity of long laboratory sparks", *Journal of Electrostatics*, V. 65, Issue 8, pp 521-526. 2007.
[2] HILL, R. D. "Analysis of irregular path of lightning channels", *Journ. of Geoph. Research*, V. 73, pp 1897-1906, 1968.
[3] IDONE, V. P.; ORVILLE, R. E.; "Channel tortuosity variation in Florida triggered lightning", *Geophysical Research Letters*, V. 15, Issue 7, p 645-648, 1988.
[4] LACERDA, M., FRITZEN, C. L., PARO, J. C., FERNANDES, W. A., "3D Simulation of Lightning Channel", *In: IX International Symposium on Lightning Protection*, pp. 51-56, Foz do Iguaçu. Brazil, 2007.
[5] LACERDA, M., FRITZEN, C. L., PARO, J. C., FERNANDES, W. A., "The Tortuosity of stochastic simulated lightning channel", In: *29th International Conference on Lightning Protection*, 2008, Uppsala.
[6] LEVINE D. M., MENEGHINI R. “Simulation of radiation from lightning return strokes: the effect of tortuosity*”. Radio Science*, Vol. 13, No. 5, 1978.
[7] LUPO G., PETRARCA C., TUCCI V., VITELLI M. “EM fields associated with lightining channels: on the effect of tortuosity and brannching”. *IEEE Trans. on EMC*, Vol. 42, No. 4, November, 2000.
[8] MACGORMAN, D. R.; RUST, D.**The electrical nature of storms**. Oxford, Oxford University, 1998. 422 p
[9] THOMAZ JR. L. D.; Dissertação MSc: “Análise da tortuosidade do canal em relâmpagos simulados em nuvens com três e quatro centros de carga”. Universidade Federal de Mato Grosso do Sul, Campo Grande, MS. 2011.

Main Author: Moacir Lacerda
Addres: UFMS CCET Unicade V -Física – Av. General Costa e Silva, sn. Bairro Universitário -79070-900
Phone: +55 67 33457035 33457540
e-mail: moacirlacerda@gmail.com cc moacir.lacerda@ufms.br